**Introduction:** The most accurate and complete discretion of the world is given by Quantum Mechanics. This theory came into existence with the failure experimental results of classical mechanics at microscopic level. Classical mechanics explains the dynamics of bodies at normal distances and properties of light based on electrodynamics. Hence, a new theory was developed to explain the properties of microscopic system with accuracy.

Quantum theory is the fundamental theory that explains the properties of matter at atomic and subatomic levels, the radiation, and the interaction of radiation with matter. This theory was developed by Max Plank, Albert Einstein, de-Broglie, Werner Heisenberg and Schrodinger. An introduction to this idea was laid by Max Plank with the exchange of energy between matter and radiation.

The description of microscopic systems is done through wave mechanics in terms of a continues wave function. The energy, momentum, position, angular momentum etc. are the dynamical variables of microscopic particle were interpreted with a wave function. This wave function satisfies the fundamental linear differential equation, the Schrodinger wave equation.

**Wave and Particle Duality (nature):**

Wave–Particle duality is the concept in quantum mechanics that every particle can be described either as a particle or wave. In order to understand the concept of dual nature, one should have the knowledge of the characteristics of waves and particles. The particle has mass, it is located at some fixed point and it can move from one place to other and it gives energy when it slow down or stopped. Thus, the particle is specified by its (i) Mass (ii) Velocity (iii) Momentum (iv) Energy. On the other hand, the wave is a disturbance in a medium and is specified by its (i) frequency (ii) wavelength (iii) phase or wave velocity (iv) amplitude (v) intensity

**De-Broglie hypothesis of mater waves (or) Wavelength of Matter Wave:**

According to de-Broglie hypothesis, a moving particle is associated with a wave which is known as de-Broglie wave whose wavelength is given by

$$\lambda = \frac{h}{m\mathrm{v}}$$
$$= \frac{h}{p}$$

Here, $h$ is the plank's constant, $m$ is the mass, $p$ is the momentum and $\mathrm{v}$ is the velocity of the wave.

If $E$ is the energy of the particle, then from quantum theory

$$E = h\nu$$
$$= \frac{hc}{\lambda} \qquad ......(1)$$

Here, $c$ is the velocity of light, $h$ is the plank's constant and $\lambda$ is wavelength.

From Einstein's mass energy relation,

$$E = mc^2 \qquad ......(2)$$

From (1) and (2),

$$mc^2 = \frac{hc}{\lambda}$$
$$\lambda = \frac{h}{mc}$$
$$= \frac{h}{p} \qquad .......(3)$$

Here, $p$ is the momentum associated with the photon.

In case of material particle of mass $m$, moving with velocity $v$ then,

$$\lambda = \frac{h}{m\text{v}}$$
$$= \frac{h}{p} \quad .....(4)$$

This equation is known as de-Broglie wave equation. From this equation if the particles are accelerated to various velocities can produce waves of various wavelengths. Higher the particle velocity the smaller is the de-Broglie wavelength.

If $E$ is the kinetic energy of the material particle then,

$$E = \frac{1}{2}m\text{v}^2$$
$$= \frac{1}{2}\frac{m^2\text{v}^2}{m}$$
$$= \frac{p^2}{2m}$$
$$p = \sqrt{2mE}$$
$$\Rightarrow m\text{v} = \sqrt{2mE} \quad ........(5)$$

The expression for de-Broglie wavelength is,

$$\lambda = \frac{h}{p}$$
$$= \frac{h}{\sqrt{2mE}} \quad .........(6)$$

**De-Broglie wavelength associated with electrons:**

If the velocity v is given to an electron accelerating it through a potential difference of $V$ volts then the work done on the electron is $eV$ which is equal to the kinetic energy of the electron. Thus,

$$\frac{1}{2}mv^2 = eV$$
$$\text{v} = \left(\frac{2eV}{m}\right)^{\frac{1}{2}} \quad ......(7)$$

Multiply both sides with $m$,

$$m\text{v} = \left(\sqrt{2meV}\right)$$
$$\lambda = \frac{h}{m\text{v}}$$
$$= \frac{h}{\sqrt{2meV}} \quad ........(8)$$

Substitute $6.62\times10^{-34}$ kg.m$^2$/s for $h$, $9.1\times10^{-31}$ kg for mass of electron $m$, $1.6\times10^{-19}$ C for charge of electron $e$ in equation 8, then

$$\lambda = \frac{h}{\sqrt{2meV}}$$
$$= \frac{1.227\text{ nm}}{\sqrt{V}} \quad .......(9)$$

**Properties of matter waves:**
Matter waves have special characteristics when compared with electromagnetic waves. The characteristics are:

(I) Matter waves are not electromagnetic waves.
(II) Smaller the mass of particle, greater is the wavelength with it,
(III) Smaller the velocity of the particle, greater is the wavelength associated with it.
(IV) When the velocity of the particle $v = 0$, then the wavelength $\lambda = \infty$, proves that matter waves are generated only by the moving particles.
(V) If $v = \infty$ then wavelength $\lambda = 0$, so the wave becomes indeterminate. That means no material particle moves with infinite velocity.
(VI) The velocity of matter waves depends up on the velocity of material particle, that is not constant while the velocity of electromagnetic wave is constant which is equal to the velocity of light.
(VII) The matter waves are produced with moving particles whether the particles are charged or uncharged.
(VIII) The velocity of matter waves is greater than the velocity of light.

$$E = h\upsilon \quad \text{and} \quad E = mc^2$$

$$\Rightarrow h\upsilon = mc^2$$

$$\upsilon = \frac{mc^2}{h}$$

The wavelength of matter waves is $\lambda = \dfrac{h}{mv}$

The matter wave velocity,

$$u = \upsilon\lambda$$

$$= \frac{mc^2}{h} \times \frac{h}{mv}$$

$$= \frac{c^2}{v}$$

As the velocity of the particle does not exceed the velocity of light hence the velocity of matter waves is greater than the velocity light.

(IX) The wave and particle aspects of moving objects can never appear together in the same experiment. Waves have particle like properties and particles have wave like properties and both are inseparable.

(X) The wave nature of matter introduces an uncertainty in the location of the position of the particle because the position of wave cannot be identified.

**Wave Packet:**

A localized wave associated with the moving particle which is formed by the superposition of number of plane waves nearby frequencies such that they add up in some region and cancel somewhere. This localized wave is called a wave packet.

The wave packet is localized and guides the particle for its motion. This is obtained according to Heisenberg Uncertainty principle because if a plane wave or infinitesimal wave is associated with the particle; either position or momentum cannot be measured simultaneously.

**Wave function:**

The mathematical representation of wave packet is the wave function $\psi$. It is represented by

$$\psi(r,t)=\int_{-\infty}^{\infty}a_k(k)e^{-i(\omega t-k.r)}$$

Here, $a_k(k)$is the amplitude of harmonic wave with wav e vector $k$and angular frequency $\psi$ of the wave function$\psi(r,t)$.

**Physical interpretation of$\psi$ :(or) Physical Interpretation of $\psi$ (or) Importance of $\psi$**

The probabilistic interpretation of wave function $\psi(x,t)$ associated with a moving particle is:

- $\psi(x,t)$ is a complex function.
- The square of the magnitude of the wave function $|\psi(x,t)|^2$ or $\psi^*\psi$ is a real quantity.
- $|\psi(x,t)|^2$ or $\psi^*\psi$ is called as probability density that is probability of finding particle at any point in space.
- $|\psi(x,t)|^2\,dV$ or $\psi^*\psi\,dV$ gives the probability of finding the particle in the elemental volume $dV=dxdydz$
- If the particle is charged particle then$\psi^*\psi$ or $|\psi|^2$ is the measure of charge density.
- If P is the total probability density then the probability of finding the particle throughout the space is,

$$P=\int\psi^*\psi\,dV$$

$$=\iiint|\psi|^2\,dxdydz$$

- Normalization condition:
  There should be certainty in finding the particle somewhere in the space. So, the total probability must be unity. Then the probability density is integrated over space is unity.

$$\int\psi^*\psi\,dV=1$$

or

$$\iiint|\psi|^2\,dxdydz=1$$

The wave function satisfying the above condition is said to be normalized.

**Limitations of wave function$\psi$ :**

- $\psi$ must be finite for all values of $x,y,z$
- $\psi$ must be single valued that is for each set of values of $x,y,z$ the wave function $\psi$ must have only one value.
- $\psi$ must be continues in all regions except where potential energy is infinite.
- $\psi$ must vanishes at the boundaries.
- $\psi$ is analytical that is it possesses continues first order derivative.
- A wave function satisfying the following condition is said to be an orthogonal wave function,

$$\int_{-\infty}^{\infty}\psi_i^*\psi_j\,dx=0 \qquad (i\neq j)$$

**Schrodinger's time independent wave equation:** In 1925 Schrodinger developed wave equation for material particles based on de-Broglie's dual (particle &wave) nature of matter. The wave function, solution of these wave equations, represents the state of the particle in any quantum mechanical system and is used to solve problems in quantum mechanics. By using wave function one can calculate variables such as position, momentum, energy of the particle.

Consider a particle of mass $m$ moving with a velocity $\mathrm{v}$ along the x-direction is associated with a wave called de-Broglie waves at any instant of time $t$. Let $\psi(x,t)$ be the wave displacement for the de-Broglie waves at any location $x$ at time $t$, then the differential equation for the wave motion in one dimensions is given by,

$$\frac{\partial^2\psi}{\partial x^2}=\frac{1}{\mathrm{v}^2}\frac{\partial^2\psi}{\partial t^2} \quad ......(1)$$ Here, $\mathrm{v}$ is the velocity of the wave.

The solution for equation (1) is

$$\psi(x,t)=\psi_0(x)e^{-i\omega t} \quad .....(2)$$

Differentiating the equation twice with respect to $t$, we get

$$\frac{\partial\psi}{\partial t}=-i\omega(\psi_0)\, e^{-i\omega i}$$

$$\frac{\partial^2\psi}{\partial t^2}=(-i\omega)(-i\omega)(\psi_0)\, e^{-i\omega i}$$

$$\frac{\partial^2\psi}{\partial t^2}=-\omega^2(\psi_0)\, e^{-i\omega i}$$

$$\frac{\partial^2\psi}{\partial t^2}=-\omega^2\psi(x,t) \quad ......(3) \quad \left(\because \psi(x,t)=(\psi_0)\, e^{-i\omega i}\right)$$

$$\Rightarrow \frac{\partial^2\psi}{\partial t^2}+\omega^2\psi=0$$

Substitute equation (3) in equation (1),

$$\Rightarrow \frac{\partial^2\psi}{\partial x^2}=-\frac{\omega^2}{\mathrm{v}^2}\psi \quad .....(4)$$

We know that,

$$\omega=2\pi\upsilon$$

$$=2\pi\frac{\mathrm{v}}{\lambda}$$

$$\frac{\omega}{\mathrm{v}}=\frac{2\pi}{\lambda} \quad ......(5)$$

Substitute equation (5) in (4) we obtain,

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} = -\frac{4\pi^2}{\lambda^2}\psi$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{4\pi^2}{\lambda^2}\psi = 0 \qquad ......(6)$$

Substitute de-Broglie wavelength $\lambda = \frac{h}{m\text{v}}$ in equation (6)

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{4\pi^2 m^2 \text{v}^2}{h^2}\psi = 0 \qquad ......(7)$$

If $E$ and $\text{V}$ are the total and potential energies of the particle then kinetic energy is,

$$KE = E - \text{V}$$

$$= \frac{1}{2} m\text{v}^2$$

$$\Rightarrow m^2\text{v}^2 = 2m(E - \text{V}) \qquad .......(8)$$

$(\text{multiply with } m \text{ on both sides})$

Substitute (8) in (7),

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0 \qquad ......(9)$$

In quantum theory we use reduced plank's constant,

$$\hbar = \frac{h}{2\pi}$$

Now the equation (9) reduced to,

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0 \qquad ......(10)$$

Extending the equation (10) for three dimensional wave,

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0 \qquad ......(11)$$

Using the Laplacian operator, $\Rightarrow \nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0 \quad ......(12)$

Equation (12) is known as Schrodinger's time independent equation.

**Schrodinger's time dependent wave equation:**

The differential equation representing one dimensional wave motion associated with a particle is,

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{\text{v}^2}\frac{\partial^2 \psi}{\partial t^2} \qquad ......(1)$$

Here, $\text{v}$ is the velocity of the wave.

The solution for equation (1) is

$$\psi(x,t) = \psi_0(x)e^{-i\omega t} \qquad .....(2)$$

Differentiating eq (2) with respect to $t$, we get

$$\frac{\partial\psi}{\partial t} = -i\omega(\psi_0)\, e^{-i\omega i}$$

$$\frac{\partial\psi}{\partial t} = -i(2\pi\upsilon)e^{-i\omega i}$$

$$= -i(2\pi\upsilon)\psi \qquad (\text{from eq.}(2))$$

$$= -2\pi i\left(\frac{E}{h}\right)\psi \qquad \left(E = h\upsilon \;\; or\; \upsilon = \frac{E}{h}\right)$$

$$= \frac{-iE}{\hbar}\psi \qquad \left(\hbar = \frac{h}{2\pi}\right)$$

$$E\psi = -\frac{\hbar}{i}\left(\frac{i}{i}\right)\frac{\partial\psi}{\partial t} \qquad (i^2 = -1)$$

$$= i\hbar\frac{\partial\psi}{\partial t} \qquad .....(3)$$

Substituting the value of E from Schrodinger's time independent wave equation, we get

$$\Rightarrow \nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

$$\Rightarrow \nabla^2\psi + \frac{2m}{\hbar^2}\left(i\hbar\frac{\partial\psi}{\partial t} - V\psi\right) = 0$$

$$\Rightarrow \frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t} \qquad ........(4)$$

This equation is known as Schrodinger's time dependent wave equation. Equation (4) can be written as

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\hat{H}\psi = \hat{E}\psi \qquad .......(5)$$

Where $\hat{H} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)$ is the Hamiltonian Operator.

And $\hat{E} = i\hbar\frac{\partial\psi}{\partial t}$ is Energy Operator

Equation (5) represents the motion of a non- relativistic material particle.

**Particle in one dimensional potential box (or) potential well (or) Energy levels of a particle enclosed in one dimensional potential box of infinite height:**

Consider a particle of mass $m$ moving along with x-axis enclosed in one dimensional potential box and the width of the box is $a$ and has elastic collisions with the walls.

The boundary conditions are,

$V(x) = 0 \quad$ for $0 \le x \le a$

$V(x) = \infty \quad$ for $0 \ge x \ge a$

To find the wave function $\psi$ of the particle with in the box of length $a$, consider the Schrodinger one dimensional time independent wave function,

$$\Rightarrow \frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0 \qquad .......(1)$$

Since the potential energy inside the well is zero that is $V = 0$ equation (1) becomes

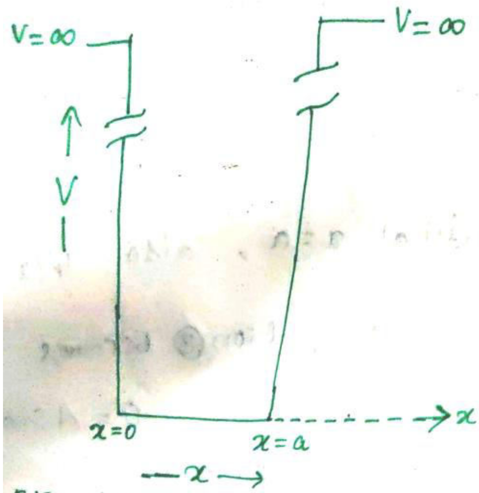


Fig: One-dimensional square well potential

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E)\psi = 0$$

Say, $\frac{2mE}{\hbar^2} = k^2$

Then the equation becomes,

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + k^2 \psi = 0 \qquad .......(2)$$

Equation (2) is a second order differential equation. So, the solution will have two have two arbitrary constants.

The solution for equation (2) is,

$$\psi(x) = A\sin kx + B\cos kx \qquad .......(3)$$

Here, A and B are arbitrary constants which can be obtained by applying boundary conditions.

(i) At $x = 0$ there is no chance for finding the particle at the walls of the box.

a. $\therefore \psi(x) = 0$

Equation (3) becomes,

$$0 = A\sin 0 + B\cos 0$$

$$\therefore B = 0 \qquad (\because \sin 0 = 0, \cos 0 = 1)$$

(ii) At $x = a$ and $\psi(x) = 0$

Equation (3) becomes

$$0 = A\sin ka + B\cos ka$$

$$\therefore A\sin ka = 0 \qquad (\because B = 0)$$

In this equation, since $B = 0$, $A \neq 0$ so,

$$\sin ka = 0$$

We know $\sin n\pi = 0$

Comparing these two equations,

$$n\pi = ka$$

$$\Rightarrow k = \frac{n\pi}{a} \qquad ......(4)$$

Substitute the values of B and k in equation (3) we get,

$$\psi(x) = A\sin\frac{n\pi}{a}.x \qquad .........(5)$$

*Normalization of wave function* $\psi$ *:*

For a one dimensional potential box of length $a$, the probability

$$P = \int_0^a |\psi|^2 dx = 1$$

$$= \int_0^a A^2 \sin^2\frac{n\pi}{a}.x = 1$$

$$\Rightarrow A^2\left(\int_0^a\left(\frac{1-\cos 2\left(\frac{n\pi}{a}\right)x}{2}\right)dx\right) = 1$$

$$\Rightarrow \frac{A^2}{2}\left(x - \sin\left(\frac{2n\pi}{a}\right)x\right)_0^a = 1$$

$$\Rightarrow \frac{A^2}{2}((a-0)-(0-0)) = 1$$

$$\Rightarrow \frac{A^2}{2}a = 1$$

$$\Rightarrow A = \sqrt{\frac{2}{a}} \qquad ......(6)$$

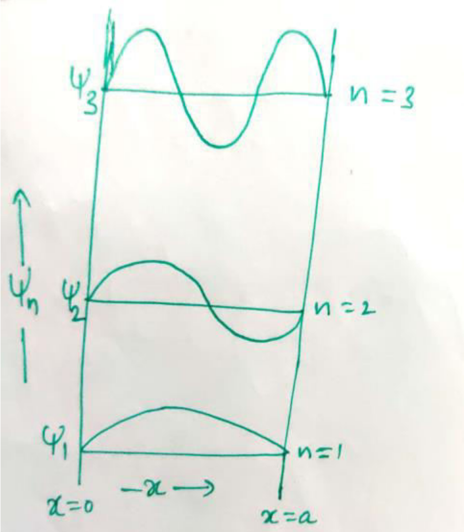


Fig2: Wave functions of a Particle in potential Box

Substitute the value of $A$ in equation (5),

$$\psi(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}\right).x \qquad .........(7)$$

This equation is known as normalized wave function. Also equation (7) gives the wave functions of the particle enclosed in infinitely deep potential well. The wave functions of the particle that is $\psi_1, \psi_2$ and $\psi_3$ corresponding to $n = 1,2$ and $3$ are shown in the figure2.

Also from equation (4),

$$k^2 = \frac{n^2\pi^2}{a^2}$$

$$\Rightarrow \frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{a^2} \qquad \left(\text{from eqation } k^2 = \frac{2mE}{\hbar^2}\right)$$

$$\Rightarrow E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

$$= \frac{n^2h^2}{8ma^2} \qquad .........(8)$$

Where, $n = 1{,}2 \text{ and } 3 \quad \text{and } \hbar = \frac{\text{h}}{2\pi}$

From equation (8) it is inside an infinitely deep potential well, the particle can have only discrete set of values of energy that is the energy of the particle is quantized. The discrete energy values are given by

$$E_1 = \frac{\pi^2\hbar^2}{2ma^2} \text{ for } n = 1$$

$$E_2 = \frac{4\pi^2\hbar^2}{2ma^2} \text{ for } n = 2$$

$$E_3 = \frac{9\pi^2\hbar^2}{2ma^2} \text{ for } n = 3$$

$$E_4 = \frac{16\pi^2\hbar^2}{2ma^2} \text{ for } n = 4$$

The energy levels are shown in the figure.

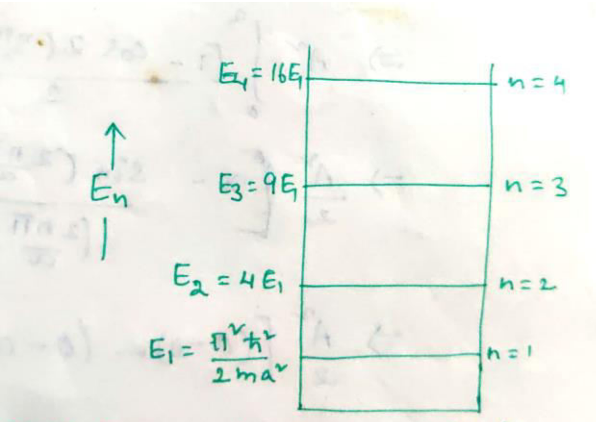


Fig 3: Energy levels of a particle inside an infinite potential well.

The probability density that is the probability of finding the particle is given by,

$$P(x) = |\psi|^2 dx$$
$$\Rightarrow P(x) = \left(\frac{2}{a}\right)\sin^2\left(\frac{n\pi}{a}\right)x$$

The probability density will be maximum when

$$\frac{n\pi x}{a} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots\ldots$$
$$\Rightarrow x = \frac{a}{2n}, \frac{2a}{3n}, \frac{5a}{2n} \ldots\ldots$$

For $n = 1$ the particle is most likely to be found at the middle of the box because $|\psi|^2$ is maximum at $x = \frac{a}{2}$

For $n = 2$, $x = \frac{a}{4}$ and $\frac{3a}{4}$ the particle is most likely to be found at $x = \frac{a}{4}$ and $\frac{3a}{4}$ here $|\psi|^2$ is maximum at these points.

Similarly at $n = 3$ $x = \frac{a}{6}, \frac{3a}{6}$ and $\frac{5a}{6}$

The variation in the probability densities is shown in the figure.

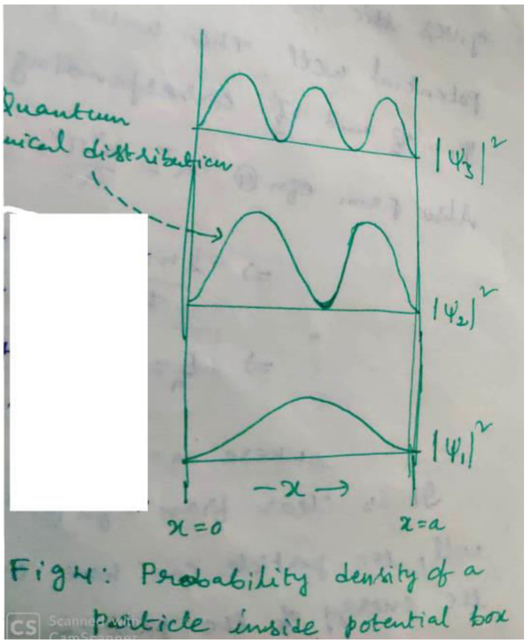


Fig4: Probability density of a particle inside potential box

**Numerical Problems on Problems on Quantum Physics**

1. **Calculate the wavelength associated with an electron raised to a potential 1600 V?**

Solution: Given that,
Potential V= 1600 V
The wavelength associated with an electron is,

$$\lambda = \frac{1.277}{\sqrt{V}}\ \text{nm}$$
$$= \frac{1.277}{\sqrt{1600}}\ \text{nm}$$
$$= 0.031\ \text{nm}$$
$$= 0.31\ \text{A}^0$$

2. **An electron is bound in one –dimensional infinite well of width $1\times10^{-10}$ m. Find the energy in the ground state and first two excited states.**

Solution: Given that

Width of potential well L $=1\times10^{-10}$ m

The energy associated with $n^{th}$ level is,

$$E_n = \frac{n^2h^2}{8mL^2}$$

Here, $h$ is Plank's constant, $m$ is mass of electron and $L$ is the width of infinite well. Substitute $6.64\times10^{-34}$ J.s for $h$, $9.1\times10^{-27}$ kg for $m$, 1 for $n$ and $1\times10^{-10}$ m for $L$ in the energy equation.

$$E_n = \frac{n^2h^2}{8mL^2}$$
$$E_1 = \frac{(1)^2\left(6.64\times10^{-34}\ \text{J.s}\right)^2}{8\left(9.1\times10^{-27}\ \text{kg}\right)\left(1\times10^{-10}\ \text{m}\right)^2}$$
$$= 0.6038\times10^{-17}\ \text{J}$$
$$= \frac{0.6038\times10^{-17}}{1.6\times10^{-19}}\ \text{eV}$$
$$= 37.73\ \text{eV}$$

The second energy state is,

$$\text{E}_2 = 4E_1$$
$$= 4(37.73\ \text{eV})$$
$$= 150.95\ \text{eV}$$

The third energy state is,

$$\text{E}_3 = 9E_1$$
$$= 9(37.73\ \text{eV})$$
$$= 339.639\ \text{eV}$$

3. **Calculate the wavelength associated with an electron of energy 2000 eV?**

Solution: Given that, the energy associated with an electron $E = 2000$ eV

Convert the energy of electron in eV to J.

$$2000\ \text{e V} = 2000\left(1.6\times10^{-19}\right)\text{J}$$

$$E = \frac{1}{2}mv^2$$

The kinetic energy is,
$$= \frac{\text{p}^2}{2m}$$
$$p = \sqrt{2mE}$$

The wavelength associated with the electron is,

$$\lambda = \frac{h}{p}$$

$$= \frac{h}{\sqrt{2mE}}$$

Here, $h$ is Plank's constant, $m$ is mass of electron and $L$ is the width of infinite well. Substitute $6.64 \times 10^{-34}$ J.s for $h$, $9.1 \times 10^{-27}$ kg for $m$, and $2000\ \text{e V} = 2000(1.6 \times 10^{-19})$ J for $E$ in the wavelength equation,

$$\lambda = \frac{h}{p}$$

$$= \frac{6.64 \times 10^{-34}\ \text{J.s}}{\sqrt{2(9.1 \times 10^{-27}\ \text{kg})(2000(1.6 \times 10^{-19}))}}$$

$$= 0.0275\ \text{nm}$$

4. **Calculate the velocity and kinetic energy of an electron of wavelength** $1.66 \times 10^{-10}$ m

Solution: Given the wavelength of electron is,

$$\lambda = 1.66 \times 10^{-10}\ \text{m}$$

Wavelength and velocity of an electron is related by the equation,

$$\lambda = \frac{h}{mv}$$

Here, $h$ is Plank's constant, $m$ is mass of electron and $v$ is velocity of electron.
The velocity of electron is,

$$v = \frac{h}{m\lambda}$$

Substitute $6.64 \times 10^{-34}$ J.s for $h$, $9.1 \times 10^{-27}$ kg for $m$ and $1.66 \times 10^{-10}$ m for $\lambda$ in the above equation,

$$v = \frac{(6.64 \times 10^{-34}\ \text{J.s})}{(9.1 \times 10^{-27}\ \text{kg})(1.66 \times 10^{-10}\ \text{m})}$$

$$= 438.9\ \text{m/s}$$

The kinetic energy of electron with the wavelength is,

$$KE = \frac{1}{2}mv^2$$

Substitute $9.1 \times 10^{-27}$ kg for $m$ and $438.9$ m/s for $v$ in the above equation,

$$KE = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(9.1 \times 10^{-31}\ \text{kg})(438.9\ \text{m/s})^2$$

$$= 8.754 \times 10^{-18}\ \text{J}$$

$$= \frac{8.754 \times 10^{-18}}{1.6 \times 10^{-19}}\ \text{eV}$$

$$= 54.71\ \text{eV}$$

**ASSIGNMENT TOPICS**

| | |
|---|---|
| 1 | (A) What are matter waves? Obtain the expression for wavelength of matter waves? [10M;CO4;Understand]<br>(B) Calculate the wavelength associated with an electron raised to a potential 1600 V. –[2M;CO4;Apply] |
| 2 | (A) Derive the de-Broglie wavelength of matter waves associated with a moving electron? Find out the wavelength of electron moving with a potential of 100V [8M;CO4;Apply]<br>(B) Discuss briefly concept of wave & particle nature. [4M;CO5;Understand] |
| 3 | (A) Derive the time independent Schrodinger wave equation?- [10M;CO4;Understand]<br>(B) Calculate the de-Broglie wavelength associated with a proton moving with a velocity of $1/20^{th}$ of the velocity of light? –[2M;CO4;Apply] |
| 4 | (A) Derive the time dependent Schrodinger wave equation? [10M;CO4;Understand]<br>(B) Calculate the de-Broglie wavelength associated with a neutron whose kinetic energy is two times the rest of the electron? [4M;CO4;Apply] |
| 5 | (A) Apply Schrodinger wave equation to the case of particle in a box and show that energies particle are quantized?— [12M;CO4;Understand]<br>(B) Calculate the wavelength associated with an electron raised to a potential of 69V? [2M;CO4;Apply] |
| 6 | (A) Explain the physical significance of wave function?–[6M;CO4;Understand]<br>(B) Calculate the wavelength associated with an electron of energy 2000 eV [6M;CO4;Apply] |
| 7 | (A)Describe the characteristic properties of matter waves- [8M;CO4;Understand]<br>(B)An electron is bound in one dimensional box of size $4\times10^{-10}$ m what will be the minimum energy?- [4M;CO4; Apply] |
| 8 | (A) Differentiate matter waves with Electromagnetic waves--[8M;CO4;Understand]<br>(B)Compute the de-Broglie wavelength associated with a cricket ball of weight 120 grams moving with a velocity of 150 Km/hour striked by Virat Kohli in a 20-20 match against Srilanka.— [4M;CO5;Apply] |
| 9 | (A) Explicit constant "A" from one dimensional potential box of length *a* of equation $\psi(x) = A\sin kx$ by using normalization of wave function $\psi$. [8M;CO4;Apply]<br>(B)Compare the ground state energy Eigen value of an electron confined in a box of length 1A° to that of a 50g golf ball confined in a box of length 50cm. [4M;CO4;Apply] |
| 10 | (A) What are the limitations to be satisfied by an acceptable wave function? –[6M;CO4;Understand]<br>(B) Compute the deBroglie wavelength associated with a cricket ball of weight 130 grams moving with a velocity of 160 Km/hour striked by Rohith Sharma in a 20-20 match against Newzeland.— [4M;CO4;Apply] |